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AUTHOR Brainerd, Charles J.
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ABSTRACT

The concept of "structure" is discussed in connection with the biological and psychological sciences and shown, through a short historical analysis, to have been subject to imprecise use. The recent "structuralist movement" in the social sciences has also tended to cloud the meaning of structure rather than to clarify it. Using Whitehead's and Russell's logic, the concept of "structure" is analyzed in terms of "relations" and "elements", and it is emphasized that neither one can be reduced to the other. The remainder of the paper considers the question of how the concept of cognitive structure enters into Piaget's global vision of intellectual development. Differences between Piaget's vaguely typological conception of cognitive structure and the analytic uses of structure as developed earlier in the paper are pointed out. The premise that "different structure=different stage" is questioned as a working hypothesis for researchers. Recent developmental studies of Piaget's stage-related reasoning skills have failed to confirm the structure-stage connection. (CS)

THE DEVELOPMENT OF IMPLICATION-REASONING IN CHILDREN AND ADOLESCENTS

Charles J. Brainerd

The experiment reported here was concerned with the development of implication-reasoning ability between middle-childhood and middle-adolescence. The experiment also was concerned with the effects of two between-subjects treatments (propositional content and propositional order) and one within-subjects within Ss treatment (type of logical inference required) on subjects' tendencies to reason in terms of formal-logical implication. Herein, the accepted meaning of "implication" is the definition ascribed to it in propositional logic, viz. a proposition A (the antecedent) is said to imply a proposition B (the consequent) if A cannot be true without B also being true. In the present paper, "implication-reasoning" is operationally defined in terms of the subjects' tendencies to affirm the four logical inferences that follow from the preceding definition of 'implication'. Given that S knows "A implies B", the following four inferences are authorized:

1. Positive inference: an inference that the conjunction of A and B (A . B) can be true.
2. Denial of the consequent: an inference that the conjunction of A and not-B (A . $\bar{B}$) cannot be true.
3. Affirmation of the antecedent: an inference that the conjunction of not-A and B ($\bar{A}$. B) can be true.
4. The contrapositive inference: an inference that the conjunction of not-A and not-B ($\bar{A}$. $\bar{B}$) can be true.

These four inference categories were employed as indexes of implication-reasoning ability.

The implication-reasoning abilities of subjects of three age levels were evaluated: 8-9 year-olds, 11-12 year-olds, and 14-15 year-olds. Although almost any developmental theory would predict improvements in general reasoning ability within this age span, Piaget in particular (e.g., Inhelder & Piaget, 1958) has predicted improvements in implication-reasoning between the ages of 8 and 15. Piaget's position has not remained entirely uncontested, however, on the basis of series of reasoning experiments (Wason, 1966, 1968) employing college Ss, Wason has concluded that Piaget's claims about the development of formal reasoning skills are unjustified. The major justification for studying three specific age levels employed in the present experiment is to provide further evidence pertaining to this controversy. Of the age groups studied, the 8-9 year-old group corresponds to Piaget's substage IIB of concrete operations; the 11-12 year-old group corresponds to Piaget's substage IIIA of formal operations; the 14-15 year-old group corresponds to Piaget's substage IIIB of formal operations. According to Piagetian theory, each succeeding substage should be characterized by an increase in implication-reasoning ability.

The semantic content of the implication-reasoning propositions also was manipulated in the present experiment. Previous research with other forms of inferential reasoning (e.g., Lefford, 1946) has indicated that the meaning or semantic content of propositions affects the validity of the inferences. While previous content manipulations have involved such things as "controversial" political topics, the present content manipulation consisted of implication-reasoning problems that differed in their ultimate reinforcement consequences.

In constructing the actual implication-reasoning problems, two well-known features of the formal implication relation were incorporated: (a) the transitivity property of implication; (b) the empirical isomorph of logical

implication, the cause-effect relation. Because implication is a transitive relation, one is authorized to conclude 'A implies C' whenever one knows either 'A implies B and B implies C' or 'B implies C and A implies B'. Because the initial propositions of any three-element implication problem (such as those employed here) may be presented in either of two possible orders, propositional order also was manipulated. Finally, because empirical cause-effect is isomorphic with logical implication, the actual problems given to the subjects involved cause-effect relations of a sort that are likely to occur in most children's lives.

Method

Subjects

As previously noted, the subjects were drawn from three age levels: third graders, sixth graders, and ninth graders. Twelve boys and 12 girls were selected from each level for a total of 72 subjects. For purposes of generality, it seemed more appropriate to study average children than either bright or dull children. Therefore, only subjects with an IQ in the 90-110 range and an academic grade-point in the 2.0-2.5 range (on a 4-point scale) were included in the present sample.

All subjects also had to meet a criterion of minimal reading ability to be included. To meet the criterion, it was necessary for the subject to read six sentences that were similar in difficulty to those he or she would encounter in the implication-reasoning problems. Only one subject (a third grader) failed to evidence minimal reading ability and he was replaced by another third grader who met the criterion. This reading pretest served as an assurance that age differences in implication-reasoning could not be attributed to age differences in reading ability.

Apparatus and Materials

A Sony 230 stereo tape recorder was used throughout the experiment. Upon entering the laboratory the subject was fitted with Sony DR-6A headphones which were not removed until he or she left the room. Instructions were prerecorded on the right channel of all tapes. The subject's responses subsequently were recorded on the left channel. Thus, all subjects heard the experimenter's instructions and comments through the right headphone, while their own responses simultaneously were being recorded on the left channel.

The other important materials employed in the experiment were white cards on which the implication-reasoning propositions were printed. All cards were three inches by five inches and the antecedent propositions of each implication-reasoning problem were typed in capital red letters on each card.

Between Subjects Treatments

Age. As noted above, three age levels were studied. Also as noted, these three age levels presumably correspond to Piaget's concrete-operational substage IIB, formal-operational substage IIIA, and formal-operational substage IIIB.

Order. The 48 propositions employed in the experiment are enumerated in Table 1. The assessment procedure focused on the extent to which the subject concluded that a cause-effect relation obtained between the propositions of columns A and C of Table 1. Obviously, there are two possible orders in which the propositions of columns A, B, and C may be conjoined to necessitate the transitive inference "A causes C", viz. "A causes B and B causes C" or "B causes C and A causes B". For convenience, the former presentation order is referred to as the forward order (FO) and the latter order is referred to as the reverse order (RO). One-half of the subjects received FO problems and the other half received RO problems.

Insert Table 1 about here

Content. An examination of the propositions in Table 1 will reveal that each implication-reasoning problem involved a single central character (either Jack or Jill). The content manipulation consisted of varying the reinforcement consequences of the cause-effect outcome for this central character. One-half of the subjects were given problems in which the transitive inference ("A causes C") involved some pleasant consequence for the central character. The other half of the Ss were given problems in which the transitive inference involved some unpleasant consequence for this central character. The pleasant outcome condition is referred to as the positive content treatment (+C) and the unpleasant outcome condition is referred to as the negative content treatment (-C). The eight +C problems appear in the top half of Table 1, while the eight -C problems appear in the bottom half of Table 1.

Within Subjects Treatment

As noted earlier, the knowledge that A imples C authorizes four valid inferences. Previous research with college subjects (e.g., Wason, 1968) suggests that these four inferences may not be of identical difficulty. Since this possibility has not been examined with children, each subject was required to make all four types of inferences in relation to each of the eight implication-reasoning problems (i.e., a total of 32 inferences in all). To determine the extent to which the subject was capable of making all four types of inferences, the experimenter asked four (randomly ordered) questions of the following general form as part of each problem:

1. Positive inference: "If A occurs, then what else will occur?" (correct answers = C or both B and C).
2. Denial of the consequent: "If A occurs, then is it possible that C won't occur?" (correct answer = no).

3. Affirmation of the antecedent: "If C occurs sometime it is possible that A didn't occur?" (correct answer = yes).
4. Contrapositive inference: "If A doesn't occur, then what else could occur?" (correct ansers = C may or may not occur or B and C may or may not occur).

The first and last questions obviously required greater extemporization than did questions 2 and 3. Whenever such a situation obtains, there exists the possibility that individual differences in motivation or anxiety may increase error variance. Hence, it seemed advisable to institute some precautionary measures to minimize the chances that failures to answer questions 1 and 4 might be due to reticence on the part of S. These precautionary measures consisted of a minimum of two promptings that were provided whenever S failed to answer (i.e., gave an, "I don't know") either the first or last question.

Randomizations

Equal numbers of subjects from the three age groups were assigned randomly to the two order conditions and to the two content conditions, with the single provision that the treatment levels be divided equally with respect to sex. From the eight factorial ways in which the implication problems might have been ordered, 24 problem orders were selected at random and randomly assigned to the subjects within each age group. Finally, the order in which the four assessment questions were posed was varied randomly for each randomly ordered problem.

Procedure

Each subject was given either the eight implication-reasoning problems appearing in the top half of Table 1 or the eight implication-reasoning

problems appearing in the bottom half of Table 1. The elements of these problems were three base propositions (columns A, B, and C of Table 1). As a means of controlling for possible age changes in short term memory, the problems were presented one-at-a-time on 3 x 5 cards and the experimenter read each one aloud. The subject was allowed to retain and reread the card during the interim during which they were asked questions concerning it.

In reading each problem, the experimenter conjoined the column A proposition with the column B proposition and the column B proposition with the column C proposition via a cause-effect relation. Following the reading of each 3 x 5 card, the experimenter assessed the extent to which the subject inferred that a cause-effect relation also obtained between the column A and C propositions (i.e., 'A causes C') via four questions of the general type noted above.

Prior to seeing the initial problem, the subject heard the following instructions: "I am going to show you some white cards one-at-a-time. On each card there is a short little story about a boy named Jack (girl named Jill). I shall read each story to you as you look at the card. I shall then ask you some questions about the story on the card. When you have answered the questions, we shall go on to a new card and to the same thing again until we are finished."

To summarize the procedural details of the experiment, the subjects from each age group were assigned randomly to either condition FO or condition FO and to either condition +C or condition -C. Next, the subjects were read the above instructions. Finally, the eight cards with the implication-bearing problems were presented and read aloud one-at-a-time; the subjects were asked four questions pertaining to each card.

Dependent Variable

The dependent measure was the subjects' answers to 32 questions (i.e., the correctness-incorrectness of the responses). The four categories of inferences were scored in the following manner:

(1) Positive inference: The subject received a 3 for a correct answer on the first try; a score of 2 for a correct answer following a single prompting; and a score of 1 for a correct answer following a second prompting.

(2) Denial of the consequent: The subject received a 2 for a correct "no" answer.

(3) Affirmation of the antecedent: The subject received a 2 for a correct "yes" answer.

(4) Contrapositive inference: The subject received a 3 for a correct answer on the first try; a score of 2 for a correct answer following a single prompting; and a score of 1 for a correct answer following a second prompting.

Since this scoring procedure established a priori differences in the absolute scores that were possible for each inference category, these absolute scores were of no use in determining the relative difficulty of each type of inference. Instead, a percentage score was calculated for each subject for each category. All subsequent analyses of the subjects implication-reasoning performances are based on such percentages rather than on absolute scores.

Results

A 3(age) x 2(order) x 2(content) x 4(type of inference) analysis of variance with repeated measures on the last factor was performed on the implication-reasoning percentages. A summary of this analysis appears in Table 2. Since a total of 15 F ratios are calculated in a four-way analysis of variance, the alpha level was set at .01 rather than the usual .05.

Insert Table 2 about here

Between Subjects Effects

It is apparent from Table 2 that the adequacy of the subjects' answers was influenced by both their age and the content of the problems. The subjects performed more adequately as they became older and they also performed more adequately if they received +C problems rather than -C problems. The order in which the subjects received the antecedent propositions did not affect the adequacy of their answers. Finally, the main effects of age and content were simple and additive with no interactive tendencies being noted.

Since three age groups were studied, the possibility of nonlinear developmental trends existed. For example the difference between third and sixth graders might have been proportionately larger than the difference between sixth and ninth graders (or vice versa). An orthogonal polynomials analysis of the age trend therefore was conducted, but only a linear trend was noted (linear $F = 22.81$, $p < .0005$; quadratic $F < 1$).

In sum, the between Ss results support the conclusion that implication-reasoning ability increases linearly across the three age groups studied and the further conclusion that the reinforcement consequences of implication problems affect the adequacy of implication-reasoning. However, the within-subject portion of the analysis revealed interactions which necessitate qualification of these two conclusions.

Within Subjects Effects

The within-subjects effect for type of question indicates that the four categories of inferences were not of equivalent difficulty. Moreover, the question by age interaction indicates that the rank ordering of inference

category difficulty was different for different age levels. Finally, the question by content interaction suggests that the content manipulation did not affect all of the forms of inference in precisely the same way.

The matched t ratios between the four inference categories appear in Table 4 and the performance means for each age level appear in Table 3. Since 24 matched t's were calculated for Table 4, only two-tailed significance levels are reported.

Insert Tables 3 and 4 about here

On the average (i.e., disregarding age), the positive inference seems to have been the least difficult. Also on the average, denial of the consequent seems to have been next in order of difficulty, while affirmation of the antecednet and contrapositive inference were the most difficult inferences. These averages are not very meaningful, however, because category difficulty varied with both age and propositional content.

While positive inference apparently is the easiest category across all three age levels, differences between the remaining three categories are evident only at the 8-9 and 11-12 year-old levels. Denial of the consequent is easier than either affirmation of the antecedent or contrapositive inference at the 8-9 year-old level and affirmation of the antecedent is easier than contrapositive inference when one considers the 11-12 year-old level. At the 14-15 year-old level, however, denial of the consequent, affirmation of the antecedent, and contrapositive inference are equally difficult.

Further post hoc analyses indicated that only three of the four inference categories improved with age: positive inferences ($p < .025$), affirmation of the antecedent ($p < .005$), and contrapositive inference ($p < .05$). Denial of

the consequent remained relatively constant in difficulty across the three age levels.

Finally, the question by content interaction indicates that the content manipulation did not affect the four forms of implication-reasoning inferences equally. Post hoc analyses revealed that the content manipulation affected only positive inferences ($p < .01$) and contrapositive inferences ($p < .01$) and contrapositive inferences ($p < .001$).

Discussion

There certainly is evidence for Piaget's claim that implication-reasoning ability improves between the ages of 8 and 15. There also is evidence for the additional claim that the content of implication-reasoning propositions contributes to the adequacy of reasoning. Neither of these results is particularly surprising, since the former would be predicted by almost any developmental theory (e.g., Inhelder & Piaget, 1958) and the latter could be predicted from experiments (e.g., Lefford, 1946) in which propositional content has affected other forms of reasoning. The within-subjects effect of type of inference and its interactions with age and content are new and interesting results, however. These three results not only necessitate qualification of the age and content effects, but they also suggest that some rethinking of the ways in which we study reasoning and its development may be called for.

Why are the four forms of inference of unequal difficulty? In particular, what accounts for the ultimate difference in difficulty (which emerges by the 14-15 year-old level) between positive inference, on the one hand, and the remaining forms of inference on the other? To simplify this question, it first should be noted that "A implies C" can be represented by four constituent conjunctions (its full disjunctive normal form):

$$A \text{ implies } C = (A \cdot C) \vee (\overline{A \cdot \bar{C}}) \vee (\bar{A} \cdot C) \vee (\bar{A} \cdot \bar{C})$$

It will be remembered from the introductory section that the first of these conjunctions $(A \cdot C)$ corresponds to positive inference. The remaining three conjunctions $(\overline{A \cdot \bar{C}}, \bar{A} \cdot C, \bar{A} \cdot \bar{C})$ correspond to denial of the consequent, affirmation of the antecedent, and contrapositive inference, respectively. Inspection of these conjunctions reveals that positive inference is the only one of the four that does not involve the use of negative information and/or the drawing of a negative conclusion. In view of the already well known inhibitory effects of negative information on concept formation (e.g., Smoke, 1932, 1933; Hovland & Weiss, 1953), a "negative information effect" seems a likely explanation for the consistent superiority of positive inference. The explanation offered here, then, is that the negative information effect that has been found repeatedly in other conceptual context probably also is operative with implication reasoning. Further support for this explanation comes from Wason's (1968) studies of deductive reasoning in college subjects. Wason concludes that college subjects use positive information more easily than negative information and draw positive conclusions more easily than negative conclusions.

The interaction of inference category with age also is interesting, because the pattern of the interaction suggests the emergence of a reasoning operation. While significant differences among the last three inference categories (denial of the consequent, affirmation of the antecedent, contrapositive inference) generally are evident at the 8-9 and 11-12 year-old levels, these forms of inference are of equivalent difficulty at the 14-15 year-old level. The clear suggestion is that the knowledge derived from A implies C is less variable and less dependent on external influences such as form-of-question by the time the subjects reach middle-adolescence. This in turn

suggests that a general reasoning skill is being consolidated between the ages of 8 and 15.

The interaction of inference category with content also must be explained. As previously noted, this interaction resulted from the fact that positive and contrapositive inferences were affected by the content manipulation but denial of the consequent and affirmation of the antecedent inferences were not. One way of explaining this finding involves first asking whether or not there are features common to positive and contrapositive inferences that might account for their differential succeptibility to the content manipulation. It turns out that there are. Here, I refer to the fact that both positive and contrapositive inferences require a certain amount of extemporization. To be judged as having given a correct positive or contrapositive inference, the subject had to produce a phrase or a sentence. In contrast, denial of the consequent and affirmation of the antecedent necessitated only a simple "yes" or "no". It seems quite reasonable that a semantic manipulation would be more likely to affect extemporaneous responses than simple agreements-disagreements (where guessing is both possible and probable).

Finally, the within-subjects main effect and its interaction with age and content suggest some additional complexities inherent in the study of the development of implication-reasoning which are not evident in Inhelder and Piaget's (1958) research. The most important of these complexities is the indication that the subjects' inferences at the two younger age levels are as much a function of the type of question posed as they are a function of developmental status. This is definitely not the impression one derives from Inhelder and Piaget. Instead, one derives that impression that implication-reasoning is a thoroughly unitary capacity whose presence can be evaluated with equal precision in a number of formal reasoning situations.

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Table 1

Implication-Reasoning Propositions

A	B	C
Jack (Jill) sweeps the kitchen floor.	Mother is very pleased.	Jack (Jill) gets all the dessert he (she) wants.
Jack (Jill) washes the dishes.	Father is very pleased.	Jack (Jill) gets 50¢.
Jack (Jill) gets all 'A's' on his (her) report card.	Both parents are very pleased.	Jack (Jill) gets to stay up later as a reward.
Jack (Jill) does well on an English test.	Teacher is happy.	Jack (Jill) gets less schoolwork to do.
Jack (Jill) mows the lawn.	Jack (Jill) works hard at something.	Jack's (Jill's) father treats him (her) to an ice cream cone.
Jack (Jill) has a birthday.	All of Jack's (Jill's) relatives come to see him (her).	Jack (Jill) gets a lot of money.
Jack (Jill) helps mother with the shopping.	Mother does not have much work to do.	Mother fixes Jack's (Jill's) favorite food for dinner.
Jack (Jill) plays a part in a school play.	Jack (Jill) does a very good job.	Jack (Jill) gets an award from the school.
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Jack (Jill) throws a rock at a window.	A window breaks.	Mother sends Jack (Jill) to bed without supper.
Jack (Jill) complains about how bad things are.	Jack's (Jill's) friends get mad.	Jack's (Jill's) friends will not speak to him (her).
Jack (Jill) plays with matches.	Jack (Jill) starts a fire in his (her) house.	Jack's (Jill's) father takes away his (her) allowance.
Jack (Jill) leaves his (her) bicycle in the driveway.	Father hits Jack's (Jill's) bike with his car.	Father takes away Jack's (Jill's) bike.

Table 1 (continued)

A	B	C
Jack (Jill) forgets something he (she) is supposed to do.	Father tells Jack (Jill) what to do.	Jack (Jill) feels ashamed.
Jack (Jill) whispers in class.	Teacher gets mad.	Jack (Jill) must stay after school.
Jack (Jill) breaks one of mother's favorite dishes.	Mother is unhappy.	Jack (Jill) cannot go outside and see his (her) friends.
Jack (Jill) doesn't come home right after school.	Jack's (Jill's) parents worry about him (her).	Jack (Jill) cannot watch television.

Table 2

Summary of Analysis of Variance of Implication-Reasoning Percentages

Source	SS	df	MS	F	P
Between Ss	69563.6	71			
Age (A)	18763.4	2	9381.7	16.50	.0005
Order (B)	260.4	1	260.4	1	NS
Content (C)	11501.7	1	11501.7	20.23	.0005
A x B	1841.0	2	920.5	1.62	NS
A x C	2334.1	2	1168.6	2.06	NS
B x C	71.9	1	71.9	1	NS
A x B x C	814.0	2	407.0	1	NS
$Error_b$	34117.9	60	568.6		
Within Ss	230677.7	216			
Question (D)	50355.1	3	16951.7	24.38	.0005
D x A	15684.4	6	2614.1	3.76	.005
D x B	1661.3	3	553.3	1	NS
D x C	16962.9	3	5654.3	8.13	.0005
D x A x B	5211.5	6	868.6	1.25	NS
D x A x C	1	6	1	1	NS
D x B x C	4736.0	3	1578.7	2.27	NS
D x A x B x C	10589.1	6	1764.9	2.54	NS
$Error_w$	125177.4	180	695.4		

Table 3

Correct Answer Percentages by Inference Category and Grade Level

Grade Level	Category 1[a]	Category 2[b]	Category 3[c]	Category 4[d]
Third Grade	74.0%	62.6%	30.9%	36.5%
Sixth Grade	81.2%	76.1%	62.3%	45.5%
Ninth Grade	90.0%	74.0%	71.5%	61.2%

[a]Positive Inference

[b]Denial of the Consequent

[c]Affirmation of the Antecedent

[d]Contrapositive Inference

Table 4

Matched t Ratios for Differences Between Inference Categories

Age Level	Category 1 vs. Category 2	Category 1 vs. Category 3	Category 1 vs. Category 4	Category 2 vs. Category 3	Category 2 vs. Category 4	Category 3 vs. Category 4
Third Grade	-.358	5.242****	4.938****	4.199****	4.066****	-.951
Sixth Grade	2.749**	2.201*	4.537****	.013	1.027	3.310***
Ninth Grade	2.721**	3.173***	4.485****	.283	1.416	1.598
Average	1.701	3.535	4.653****	1.498	2.170*	1.650

*p .05

**p .025

***p .01

****p .0025